

# Graph Algorithms - Problem Set 1

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## 1 Algorithm Design by Induction

1. Prove by induction that trees are always bipartite.
2. Let  $G = (V, E)$  be a binary tree. The **distance** between two vertices in  $G$  is the length of the path connecting these two vertices (neighbors have distance 1). The diameter of  $G$  is the maximal distance over all pairs of vertices. Design a linear-time algorithm to find the diameter of a given tree.
3. Let  $G = (V, E)$  be a binary tree with  $n$  vertices. We want to construct an  $n \times n$  matrix whose  $ij$ th entry is equal to the distance between  $v_i$  and  $v_j$  (Since the tree is undirected, the matrix will be symmetric). Design an  $O(n^2)$  algorithm to construct such a matrix for a tree that is given in the adjacency-list representation.
4. Let  $G = (V, E)$  be directed graph. The **celebrity** vertex in  $G$ , is a vertex  $v$  where for each vertex  $w \in V$  ( $w \neq v$ ) we have  $(w, v) \in E$  and  $(v, w) \notin E$ . Design an  $O(n)$  time algorithm to find a celebrity vertex in  $G$ , or determine that there is no such a vertex in  $G$ .